Using Definition of Permeability Tactfully to Solve Multi-medium Eddy Current Problems Involving Movement by RBF

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This paper presents a new hybrid radial basis function (RBF) collocation method developed in our previous work to solve multi-medium problems with moving conductor. In order to avoid the reset of the sub-domains and nodes at different steps, the magnetic field is divided into three kinds of fields generated respectively by the excitation current, eddy current and magnetizing current based on the definition of permeability in multi-medium problems. With the movement of medium, the magnetizing current and eddy current are also moving. Moving coordinate systems in which the fields generated by magnetizing current and eddy current are calculated will be used, so the movingmedium is relatively static compared with moving coordinate systems. Hence, there is no need to refresh the sub-domains and nodes, and the proposed method is useful to solve multi-medium eddy current problems involving movement. Two numerical examples are computed to verify the method.

***Index Terms*—Magnetic fields, permeability, numerical analysis, eddy currents, radial basis function.**

I. INTRODUCTION

Using RBF collocation method to solve electromagnetic problem has been proved as an effective method [1]-[3]. However, it's not very suitable for multi-medium problem [4], [5], especially the moving medium [6], [7]. As a universal moving conductor problem, the moving domain is different medium compared with the no-motion domain, but each of the domains is homogeneous medium respectively. Therefore, how to make the utilization of RBF collocation method for multi-medium moving conductor problem is more important and has great practical significance.

We introduced a new RBF collocation method in [8]. The new approach divides computational domain into a series of sub-domains and uses the point interpolation based on RBF to obtain the shape functions respectively. Then, each separate domain is taken as elements of the Galerkin finite element method (FEM) to approximate the solutions of entire computational area. The new method can be used to deal with multi-medium problem, and the coefficient matrix becomes sparse. Nevertheless, for multi-medium moving conductor problem, the emergence of new difficulty is that there must be nodes set on the boundary between two domains of different medium. Thus, with the movement of the medium, the whole nodes and sub-domains must be reset, and it makes the new RBF collocation method difficult to implement in this kind of problem.

To overcome this difficulty, we came up with a very clever way from characteristics of electromagnetic problems, the definition of permeability. As we know, the permeability comes from the magnetizing current, and the field can be divided into two kinds of fields generated respectively by the excitation current and magnetizing current. Using field superposition principle, the field of magnetizing current moving together with the medium would be calculated in moving coordinate systems and the excitation current field in stationary coordinate systems, using two separate sets of nodes to avoid redistribution of nodes. To verify the proposed method, we compute a multi-medium static field problem to prove the correctness of using definition of permeability to decomposition field. And Team Workshop 28 problem [9] as a multi-medium moving conductor problem will also be calculated.

II. IMPROVED SUPERPOSITION HYBRID RBF METHOD

A. Decomposition field based on definition of permeability

Consider a simple 2-D static magnetic field problem composed of air sub-domain Ω_1 and other medium sub-domain Ω_2 . The governing equations can be expressed as

$$\begin{aligned}
&\nabla^2 A = -\mu_0 J_s && in\Omega_1\\
&\nabla^2 A = -\mu_1\mu_0 J_s = 0 && in\Omega_2\\
&B(A) = 0 && on\partial\Omega\\
&A_1 = A_2 && on\partial\Omega_1 \cap \partial\Omega_2\\
&\mu_1\mu_0 \frac{\partial A_1}{\partial n} = \mu_0 \frac{\partial A_2}{\partial n} && on\partial\Omega_1 \cap \partial\Omega_2
\end{aligned} \tag{1}$$

where J_s is the excitation current. μ_1 is the relative permeability of the sub-domain Ω_2 . A, A_1, A_2 are magnetic vector potential, and $B(\cdot)$ is the boundary operator.

Using definition of permeability and field superposition principle, the magnetic field is considered to be an addition of two fields $A - A_0$, A_0 , and the governing equations should be restructured. The field $A - A_0$ is

$$\begin{aligned}
&\nabla^2 (A - A_0) = 0 && in\Omega_1\\
&\nabla^2 (A - A_0) = -\mu_1\mu_0 J_s + \mu_0 J_s = 0 && in\Omega_2\\
&B(A - A_0) = 0 && on\partial\Omega\\
&A_1 - A_0 = A_2 - A_0 && on\partial\Omega_1 \cap \partial\Omega_2\\
&\mu_1\mu_0 \frac{\partial (A_1 - A_0)}{\partial n} - \mu_0 \frac{\partial (A_2 - A_0)}{\partial n} && \\
&\quad = \mu_0 \frac{\partial A_0}{\partial n} - \mu_1\mu_0 \frac{\partial A_0}{\partial n} && on\partial\Omega_1 \cap \partial\Omega_2
\end{aligned} \tag{3}$$

And the field A_0 is

$$\begin{aligned} \nabla^2 A_0 &= -\mu_0 J_s \qquad && in\Omega \\ B(A_0) &= 0 && on\partial\Omega \end{aligned} \tag{2}$$

Mathematically, the reconstruction of the governing equation is based on linear superposition principle of partial differential equations. In physics, A_0 and $A-A_0$ are formed by the excitation current and magnetizing currentre respectively. According to (3), $\mu_0 \dfrac{\partial A_0}{\partial n} - \mu_1 \mu_0 \dfrac{\partial A_0}{\partial n}$ comes from the magnetizing current distribution on the interface. Extending to the general problem of multi-media with no air medium, we can also make a similar decomposition. In moving medium, the A_0 will be calculated in stationary coordinate systems. Meanwhile, the governing equations of $A-A_0$ contain boundary condition of the interface between different medium, which is stationary relative to moving coordinate systems. Therefore, calculation process does not need to reset the nodes in this coordinate.

B. *Moving conductor eddy current problem analysis*

Consider the different permeability of moving conductor with eddy current and no-motion domain. The magnetic field is an addition of three fields, A_s generated by exciting current J_s, A_m generated by magnetizing current J_m, and A_e generated by eddy current J_e, in moving conductor eddy currentproblems. Hence, the field is expressed as

$$A = A_s + A_m + A_e \tag{5}$$

A_m should be computed in moving coordinatesystems, the same with A_e, so the implementation process is similar to a homogeneous and isotropic electromagnetic system[6]. The detail processing of the improved superposition RBF collocation method to solve this problem will be presented in full paper.

III. Numerical Examples

A. *Multi-medium static field problem*

In order to verify the new field superposition principle, we calculate a multi-medium static field problem.

According to the proposed method, we divided the magnetic field into two parts, so the fields were calculated respectively. Fig. 1 and Fig. 2 show the results by RBF method. Combining the two solutions, we got the total magneticfield as in Fig 3. Then, we also obtain the result from FEM software (Fig. 4).It can be seen that the solution by the novel approach is very close to the FEM solution.

B. *Team Workshop 28 problem*

We also calculate a moving conductor eddy current problem of Team Workshop 28 problem. Fig. 5 shows the result of levitation height from new method compared with measured value. More detail will be presented in full paper.

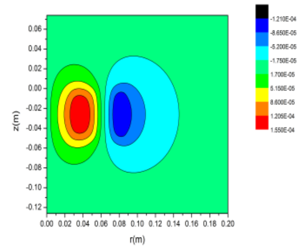

Fig. 1.The field of exciting current by RBF

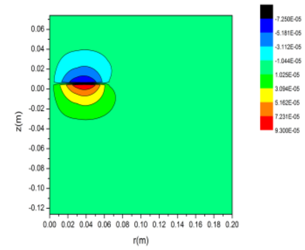

Fig. 2. The field of magnetizing current by RBF

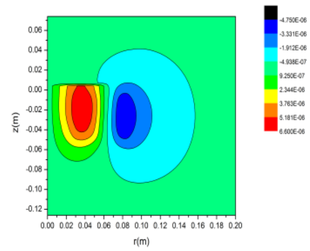

Fig. 3.The superposition result of two fields

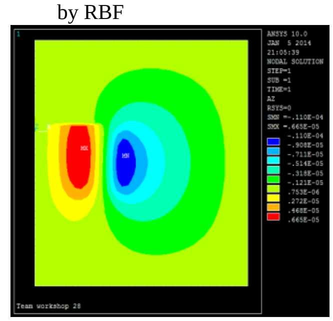

Fig. 4. The field solved by FEM directly

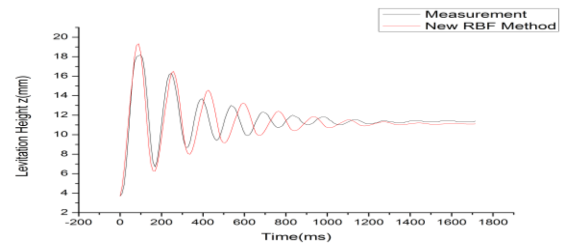

Fig.5. Comparison between the measured (solid line) and the computed levitation height (broken line) by RBF

References

[1] E. J. Kansa, "Multiquadric - a scattered data approximation scheme withapplications to computational fluid dynamics," *Comput. Math. Appl.*, vol.19, pp. 147–161, 1990.

[2] S.L. Ho, S. Yang, J.M. Machado, and H.C. Wong, "Application of a meshless method in electromagnetics," *IEEE Trans. Magn.*, vol. 37, no.5, pp. 3198-3202, 2001.

[3] R. Das, and D. A. Lowther. "Magnetic field computation using compact support radial basis," *IEEE Trans. Magn.*, vol.47, no.5, pp.1362-1365,2011.

[4] J.H. Jung. "A note on the Gibbs phenomenon with multiquadric radial basis functions," *Applied Numerical Mathematics*, vol.57,pp.213-229, 2007.

[5] J.H. Jung, V.R. Durante. "An iterative adaptive multiquadric radial basis function method for the detection of local jump discontinuities,". *Applied Numerical Mathematics*, vol.59, no.7, pp.1449-1466, 2009.

[6] X. M. Chen, G. Lei, G. Y. Yang, K.R. Shao, Y. G. Guo, J. G. Zhu, J. D. Lavers, "Using improved domain decomposition method and radial basis functions to determine electromagnetic fields near material interfaces," *IEEE Trans. Magn.*, vol. 48, no.2, pp.199-202, 2012.

[7] G. Y. Yang, et al, "A novel superposition RBF collocation method to solve moving conductor eddy current problems," *IEEE Trans. Magn.*, vol.45, no.10, pp.3977-3980, 2009.

[8] Y. Zou, G. Lei, K. R. Shao,Y. G. Guo, J. G. Zhu, and X. M. Chen, "Hybrid approach of radial basis function and finite element method for electromagnetic problems," *IEEE Trans. Magn.*, DOI 10.1109/TMAG.2014.235437.

[9] Karl Het et al, "Description of TEAM workshop problem 28: An electrodynamic levitation device," *in Proc. 6th Round,TEAM Workshop*, Rio de Janeiro, Brazil, 1999.